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Crystallographic patterns in Philippine indigenous textiles

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The aim of this study was to analyze a representative sample of Philippine indigenous textiles in order to capture the range of symmetries and color symmetries present. This paper examines the existence of symmetries in finite designs, and classifies the plane-group and frieze-group symmetry types of the repeated patterns in woven textiles. The tendency of a particular symmetry to be more or less common than another can indicate relationships between the symmetries and the weaving technique or the culture that produced them. This paper will also examine designs and patterns with color symmetry found in these textiles. The sample consisted of 588 repeated patterns and finite designs in textiles (389 plane, 166 frieze and 33 finite) culled from well known museums in the Philippines, personal collections of scholars, existing literature on Philippine textiles and field visits.

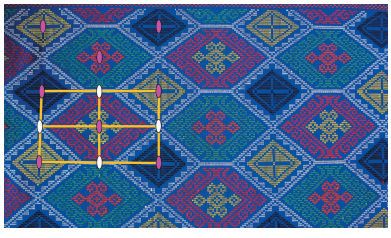
1. Introduction

Philippine woven textiles offer a glimpse into the rich cultural heritage of the Filipino people. From the northernmost parts of Luzon to the southern edges of the Sulu archipelago, the extensive range and quality of Philippine textiles are beyond question (Fig. 1). Geography and history have influenced the evolution of textile arts among the indigenous communities.

In northern Luzon, the Cordillera people, protected in part by their terrain, resisted the Spanish conquerors. Consequently, there has been more persistence of traditional arts, values and cultural identities in these areas than in the lowlands. The traditional blankets produced by the Tinguians and the Bontocs from the Cordillera region in northern Luzon and the *inabel* cotton weaves of the Ilocanos in the northern region of the Philippines offer evidence of the strong weaving tradition in these areas.

In the south, the rivalry between the Spanish and the Muslims was important in the preservation of indigenous culture. The Muslims resisted the colonization of Mindanao and were able to prevent the destruction of native arts and crafts, in contrast to what occurred in lowland Luzon and the Visayas. Their textiles include the *malong* or 'tubular garment' of the Maranaos, the *pisyabit* or 'head cover' of the Tausugs, and the *saputangan* or 'head cloth' of the Yakans. Mindanao is also home to indigenous non-Muslim communities with their own weaving traditions; their textiles include the *t'nalak* or tie-dyed fabric of the T'bolis, as well as the woven cloths of the Bagobos.

In this work, we study the existence of symmetries in finite designs and classify the plane-group and frieze-group



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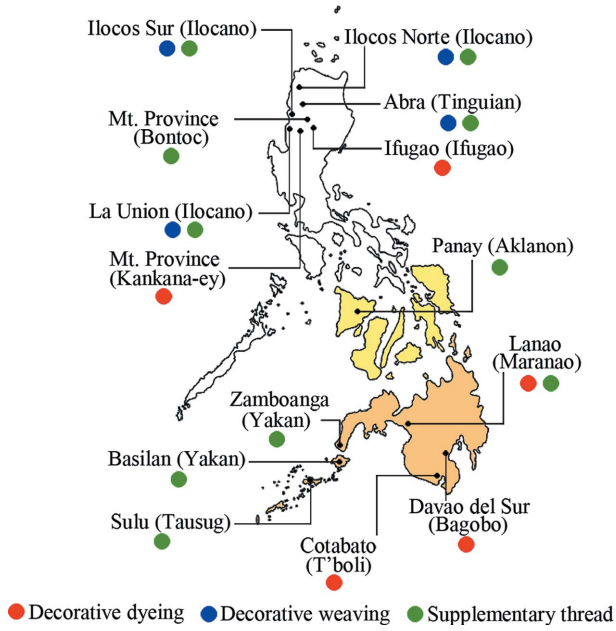


Figure 1
Weaving communities in the Philippines. The island groups of Luzon, Visayas and Mindanao are the white, light yellow and light orange colored regions, respectively. Red, blue and green dots are used to denote decorative dyeing, decorative weaving and supplementary thread weaving techniques, respectively.

symmetry types of the repeated patterns in woven textiles from indigenous communities in the Philippines. This paper will also discuss the designs and patterns with color symmetry present in the textiles. We consider the use of colors in the motifs and investigate the associated color groups of the resulting designs and patterns.

The outline of the paper is as follows: §2 presents the materials and methods used in this study, §3 details the mathematical basis for the analysis of the textiles, §4 presents the different weaving techniques employed by the indigenous communities in the Philippines, §5 discusses the symmetry group structures of the finite designs and repeated patterns present in representative textiles arising from each weaving technique, §6 explains the color symmetries present in textiles discussed previously in §5, and finally §7 provides a summary of the work.

2. Materials and methods

This study is focused on woven textiles (excluding embroidered or beaded textiles), specifically the analysis of the frieze patterns, plane patterns and finite designs of these textiles. In our study, we adapt the methodology of Washburn & Crowe (1988). A symmetrical figure within the plane is called a 'pattern' or 'design'; the terms 'pattern' or 'repeated pattern' (for emphasis) are reserved for designs with translational symmetry. A pattern is referred to as a 'frieze' if it has translational symmetries in only one direction, whereas a 'plane pattern' has symmetries that include two linearly independent translations; if a design does not have any translational symmetry, then it is referred to as a 'finite design'.

The aim of this study was to analyze a representative sample that can capture the range of symmetries and color symmetries in Philippine textiles. To achieve this, we examined samples from well known museums in the Philippines (National Museum, Yuchengco Museum, Vargas Museum, Museo Kordilyera) and private textile collections of scholars (Quintos, Salvador-Amores, Respicio), as well as the collections of the National Commission for Culture and the Arts (NCCA) and the Philippine Textile Council. We also examined the existing literature on Philippine textiles (Guatlo, 2013; Pastor-Roces, 1991; Paterno *et al.*, 2001; Respicio, 2003, 2014; Rubinstein, 1989) and expanded the sample through field visits to indigenous communities (T'boli and Yakan villages), loom weavers in the Ilocos region (Nagbacalan Loomweavers Multi-purpose Cooperative) and festivals (Dayao Festival and NCCA International Festival of Extraordinary Textiles). From these sources, a representative sample of textiles exhibiting patterns of various symmetries and color symmetries was obtained for each indigenous community.

3. Mathematical considerations

The mathematical study of a finite design or repeated pattern in a cultural ornament such as a textile entails investigation of its symmetries. An isometry of the plane (translation, rotation, reflection or glide reflection), which is a distance preserving transformation that maps a finite design or repeated pattern onto itself, is called a 'symmetry' of the respective design or pattern.

A finite design does not have translational symmetry (and consequently, has no glide reflections) and can have reflection and/or rotational symmetry. A finite design is of cyclic symmetry group type C_r if it has r-fold rotational symmetry and no reflection symmetry. On the other hand, a finite design is of dihedral symmetry group type D_r if it has reflection symmetry as well as r-fold rotational symmetry. Note that D_1 is the symmetry group of designs with bilateral symmetry only.

Frieze patterns and plane patterns are classified on the basis of the symmetries of the pattern. There are seven symmetry group types for frieze patterns, called 'frieze groups', and 17 symmetry group types for plane patterns, called 'plane groups'. Here we adopt the standard nomenclature for frieze and plane groups given by Kopský & Litvin (2002) and Hahn (2005), respectively.

In the figures that supplement our discussion of the symmetry groups of the repeated patterns, we provide the lattice units of the symmetry groups. The centers of rotation are marked by ellipses, mirror reflection axes by bold lines and glide reflection axes by dashed lines.

4. Weaving techniques

Weaving is done on a loom. The warp threads run vertically from the warp beam towards the weaver while the weft threads run perpendicular to the warp threads, that is, from left to right or *vice versa*, in relation to the weaver. Philippine

weaving techniques may be classified into three broad categories to be explained further in §4.1–4.3: (i) decorative dyeing, (ii) decorative weaving and (iii) supplementary thread techniques. Fig. 1 shows the different techniques employed in traditional weaving areas of the Philippines. Of the 588 finite designs and repeated patterns examined, 138 were woven using the decorative dyeing technique, 59 using decorative weaving techniques and 391 using the supplementary thread technique.

4.1. Decorative dyeing

Ikat is a term for describing the decorative dyeing technique. The process involves binding or knotting several threads together at certain intervals and then submerging the entire mass of threads in a dye. The knotted portions will resist the dye, from which the pattern may be seen. Some knotted threads may be unraveled and the whole mass soaked once again in a different dye to produce a more complex pattern. This process may be repeated several times. The *ikat* used in the Philippines is primarily warp *ikat*. This means that it is the warp rather than the weft threads that are soaked in dye, so that a pattern emerges when weft threads are interwoven through the dyed warp threads.

The T'bolis from Lake Sebu, Cotabato, in southern Mindanao, use wild abaca thread to produce *ikat* patterns, usually colored black, red and natural beige, the original color of the abaca leaves (Fig. 2). The warp threads are secured on a frame and portions are knotted to resist the dye. These threads are submerged in black dye and dried. Next, the areas that are to be colored red are untied and the threads boiled in red liquid. The rest of the knotted portions are then untied, and these become the areas that appear in the natural color of the abaca leaves. Weft threads are then interwoven perpendicular to the patterned warp threads to produce the cloth (Fig. 3*a*). The *ikat* cloth is referred to by the T'bolis as the *t'nalak*. The T'bolis weave the *t'nalak* relying on a mental image of the patterns, which are passed on from mother to daughter, or bestowed upon the weaver by *Fu Dalu*, the spirit of the abaca, through dreams (Paterno *et al.*, 2001). As such, the *t'nalak* is a prized possession used by the T'bolis on special occasions such as births, weddings or funerals.

The Bagobos, indigenous people from the Davao region in southern Mindanao, and the Ifugaos and Kankana-eys from

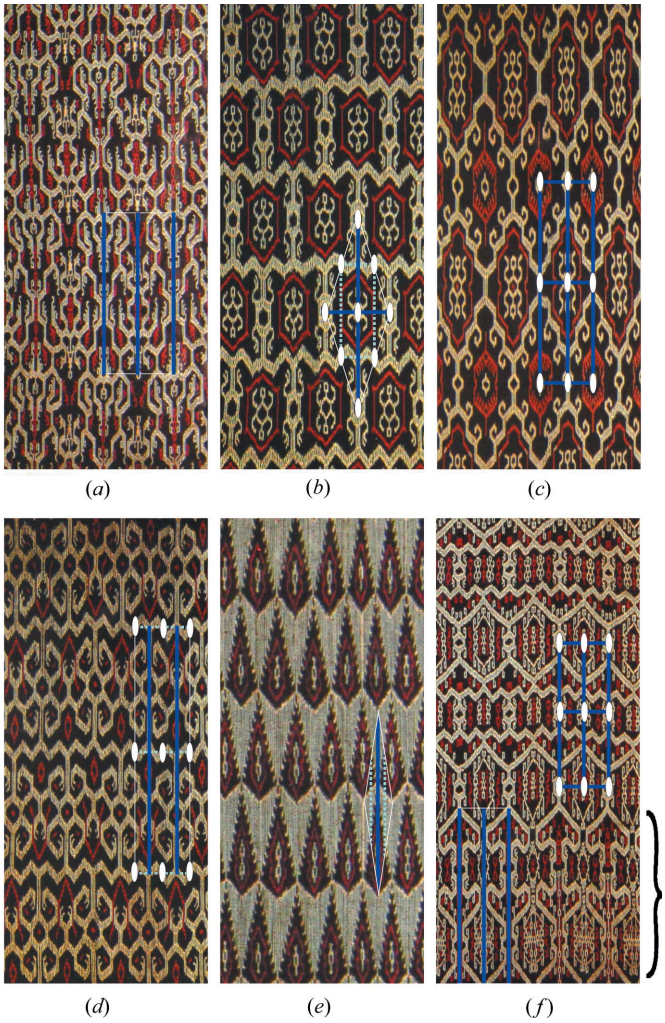


Figure 2
T'nalak with symmetry groups (*a*) *p*1*m*1, (*b*) *c*2*mm*, (*c*) *p*2*mm*, (*d*) *p*2*mg* and (*e*) *c*1*m*1. In (*f*), a *t'nalak* with primary design and symmetry group *p*2*mm*, bordered below by a frieze with symmetry group *p*1*m*1 (marked by brace). Images from Paterno *et al.* (2001) (photographed by Neil Oshima).



Figure 3
T'boli weavers. (*a*) *Agustin Sudaw* (photographed by Ma. Louise Antonette De Las Peñas, July 2014, Manila, Metro Manila); (*b*) *Charlie Dulay* (photographed by I. Castrillo, September 2017, General Santos City, South Cotabato).

the Cordillera region in northern Luzon also employ the warp *ikat*. Fig. 4 shows a blanket woven by the Bagobos, displaying crocodile motifs. The crocodile or 'big lizard' is a powerful motif used by the weaving communities among the Bagobos and Ifugaos in their tie-dyed fabrics; amphibious forms are regarded as vehicles of spirits traveling the world and beyond (Respicio, 2003).

The Maranaos from southwestern Mindanao, on the other hand, employ the weft *ikat* technique in their weaving, where the weft threads are soaked in dye. An example of the Maranao art is the *malong* shown in Fig. 5.

4.2. Decorative weaving

In decorative weaving techniques, the patterns are created not through dyeing but through the weaving process itself. The simplest among these is the plain weave, where the weft passes over and under every other warp thread (one-over-one-under). This basic weave is common among all weaving communities. Plaids or stripes are the more familiar forms of decorative weaving, and the *hablon* of the Hiligaynons, in Iloilo, western Visayas, demonstrates this kind of weave. It is usually used for the *patadyong*, the Visayan wrap-around skirt. The result of a one-over-one-under weave is a fabric whose back pattern is the 'negative' (with respect to color) of the front pattern. As such, it is sometimes called a 'face-to-face weave'.

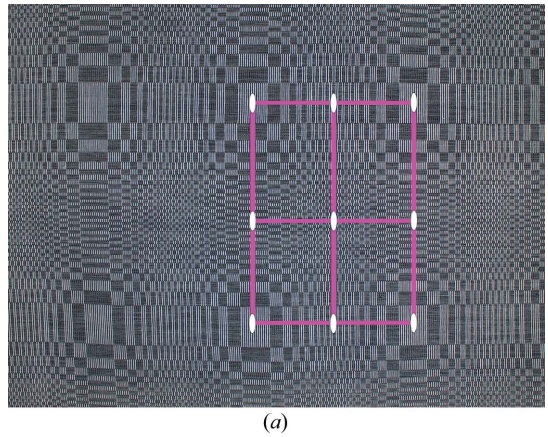
More variety can be achieved by setting differently colored warp threads on the loom in irregular intervals or by changing the color of the weft threads. One notable example is the *binakul* pattern from northern Luzon (Fig. 6); the yarns are arranged in such a way that, when the weft yarns interlock with the warp yarns, squares and rectangles of varying sizes are formed. Two hues of yarn are used (positive white and negative black, blue or deep red), and the result is an optical pattern giving an illusion of swirling circles or ripples, intended to cause dizziness to evil spirits. This kind of decorative weave technique is achieved using a pedal frame loom (Fig. 7) and is common among the weaving communities in the Ilocos region and Abra in northern Luzon.



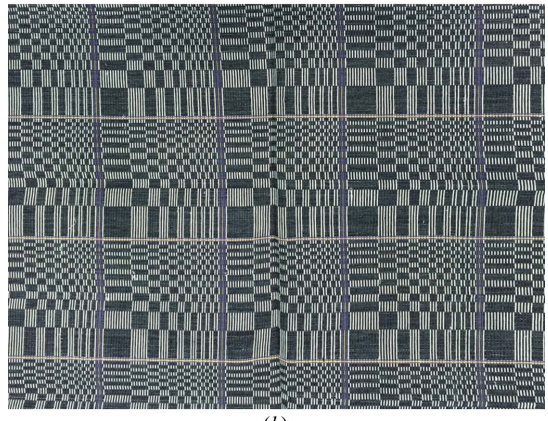
Figure 4
Warp *ikat* woven blanket from the Bagobos showing friezes with symmetry groups *p*1*m*1 and *p*2*mm*. The bottom frieze shows a crocodile design. (*Ikat from the National Museum collection*, photographed by Debbie Marie Verzosa, May 2014, Manila, Metro Manila.)



Figure 5
Malong in weft *ikat* weave from the Maranaos, showing friezes with symmetry groups *p*2*mm* and *p*1*m*1. (*Malong from the F. Quintos collection*, photographed by Ma. Louise Antonette De Las Peñas, September 2014, Quezon City, Metro Manila.)



(*a*)



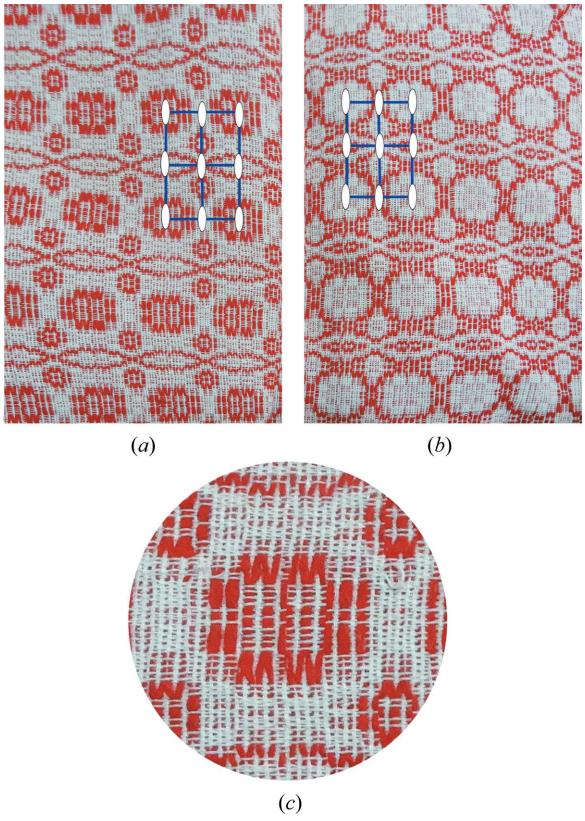
(*b*)

Figure 6
Two variants of *binakul*, the (*a*) optical and (*b*) quarter circle patterns with symmetry groups *p*2*mm* and *p*1, respectively. (*Binakul from the Phillipine Textile Council collection*, photographed by Agnes Garciano, May 2014, Manila, Metro Manila.)

Another option to achieve variety is to make the warp 'float' or skip over two or more weft threads, or to make the weft float over two or more warp threads (Fig. 8*b*). These techniques are called warp float and weft float, respectively.



Figure 7
Weaver using the pedal loom. (Photographed by Ma. Louise Antonette De Las Peñas, May 2015, Paoay, Ilocos Norte.)

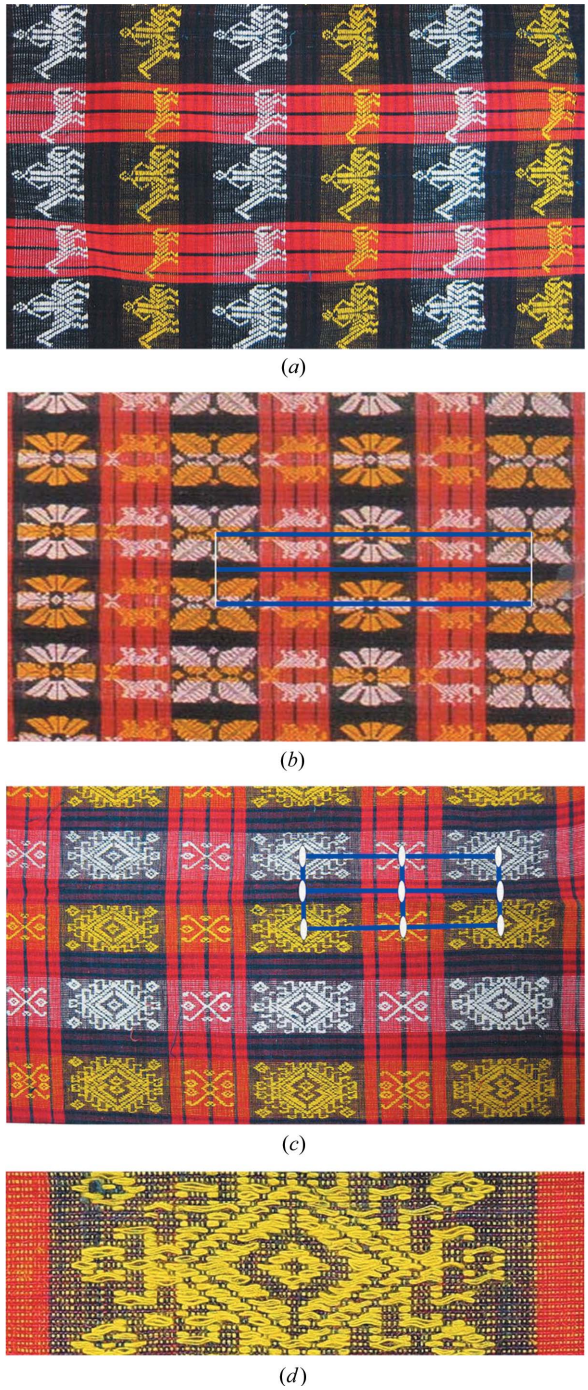


(*a*) (*b*) (*c*)

Figure 8
Textile woven using the decorative weft float technique. Patterns have symmetry group *p*2*mm*. Photographs show (*a*) the back and (*b*) the front; (*c*) detail view of the back side of the textile. (*Textile from the Nagbacalan Loomers Multi-purpose Cooperative collection*, photographed by Ma. Louise Antonette De Las Peñas, May 2015, Paoay, Ilocos Norte.)

4.3. Supplementary thread

Supplementary warp or weft techniques involve the insertion of additional extra warps or wefts to create designs. These



(*a*) (*b*) (*c*) (*d*)

Figure 9
Tinguian blankets woven in *pinilian* style with symmetry groups (*a*) *p*1, (*b*) *p*1*m*1 and (*c*) *p*2*mm*; (*d*) supplementary weft configuration employed to create a motif of the blanket in (*c*). [(*a*), (*c*) *Pinilian textiles from the F. Quintos collection*, photographed by Ma. Louise Antonette De Las Peñas, September 2014, Quezon City, Metro Manila; (*b*) Rubinstein (1989).]

threads are supplementary or extra in the sense that if they were removed the woven cloth would still be complete, though without the original pattern. In the supplementary weft technique, the additional weft threads are made to float over warp threads in a planned sequence to create the design or pattern. If the supplementary threads extend from edge to edge, they are 'continuous', but if they only go back and forth in small areas where they are needed, they are 'discontinuous'.



Figure 10
Tinguian blankets in *dinapat* style: (*a*) frieze with symmetry group *p*2*mm* and (*b*) planar pattern with symmetry group *p*2*mg*. (*c*) Detail view of (*b*). [(*a*) Rubinstein (1989); (*b*) *dinapat textile from the F. Quintos collection*, photographed by Ma. Louise Antonette De Las Peñas, September 2014, Quezon City, Metro Manila.]

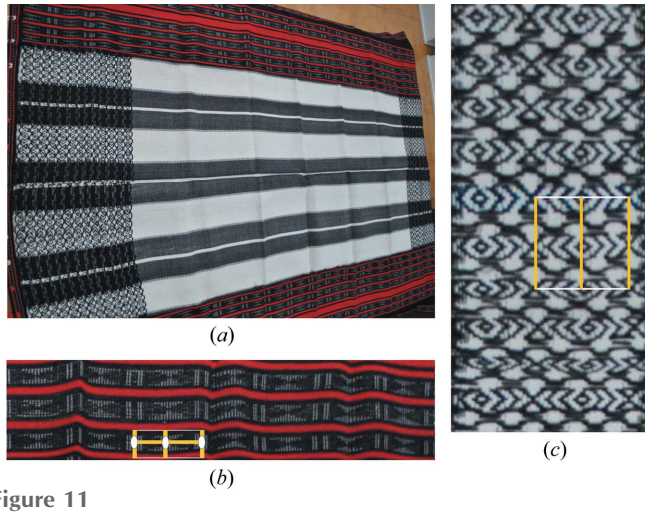


Figure 11
(*a*) *Pinagpagan*; (*b*) the friezes at the top and bottom have symmetry group *p*2*mm*; (*c*) the central panel displays a planar pattern with symmetry group *p*1*m*1 at the sides. [*Pinagpagan* (from De Las Peñas & Salvador-Amores, 2016), photographed by Dr Analyn Salvador-Amores, 2014, Baguio City, Benguet.]

4.3.1. Continuous supplementary weft. For the Tinguians and Bontocs in northern Luzon, the most common weaving technique is the continuous supplementary weft, which is evident in their blankets (Figs. 9–11). Most of the Tinguian blankets are woven using the *pinilian* (Fig. 9) or *dinapat* (Fig. 10) styles. These depict flora, leaves, human figures or animals; a common motif is the rider and horse (Fig. 9*a*). These motifs are highly valued, and the blankets are often used for funeral ceremonies. *Pinilian* means 'separated' or 'divided', referring to the checkerboard-like grid style of laying out the motifs. Note in Fig. 9(*d*) how a motif of the repeated pattern in Fig. 9(*c*) is created by the insertion of weft threads, producing 15 different horizontal weft configurations that are employed in a repeated sequence. Each weft configuration floats from left to right at repeated intervals. Meanwhile, *dinapat* denotes 'full' or 'total', referring to the repeated pattern that occupies the whole blanket. The *dinapat* blanket in Fig. 10(*b*) shows that, even without varying the breadth of the warp and weft yarns, patterns other than checkered or plaid can be produced, *i.e.* the manner in which the weft yarns are inserted produces red and white isosceles right-angled triangles.

Fig. 11(*a*) shows a *pinagpagan*, a typical funerary blanket of the Bontocs also woven using the continuous supplementary

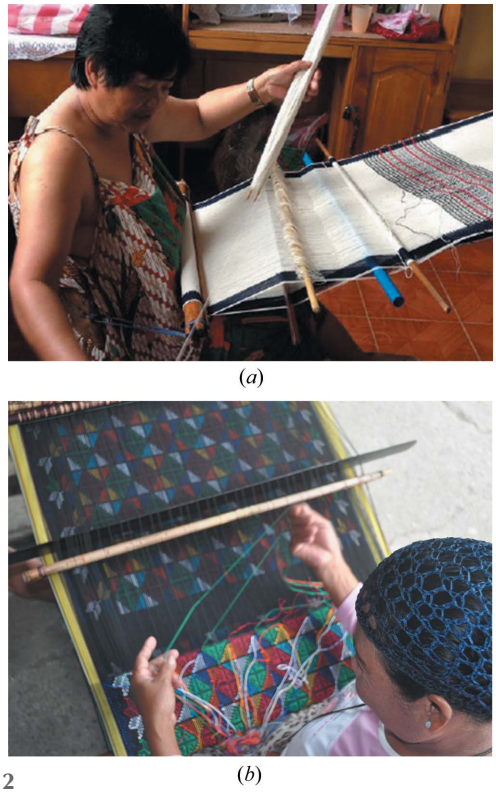


Figure 12
Weavers employing the supplementary weave technique. (*a*) Bontoc (continuous) and (*b*) Yakan (discontinuous). [(*a*) *Bontoc weaver* (from De Las Peñas & Salvador-Amores, 2016), photographed by Dr Analyn Salvador-Amores, 2014, Baguio City, Benguet; (*b*) *Yakan weaver*, photographed by Debbie Marie Verzosa, April 2015, Yakan village, Zamboanga City.]



(*a*) (*b*)

Figure 13
(*a*) Yakan textile showing the *palipattang* pattern consisting of friezes; (*b*) a representative frieze with symmetry group *p*2*mm* (lattice unit with white axes of reflection and centers of rotation) and color group *p*2*mm* (lattice unit with blue axes of reflection and pink centers of rotation). (*Textile with palipattang*, photographed by Debbie Marie Verzosa, April 2015, Yakan village, Zamboanga City.)

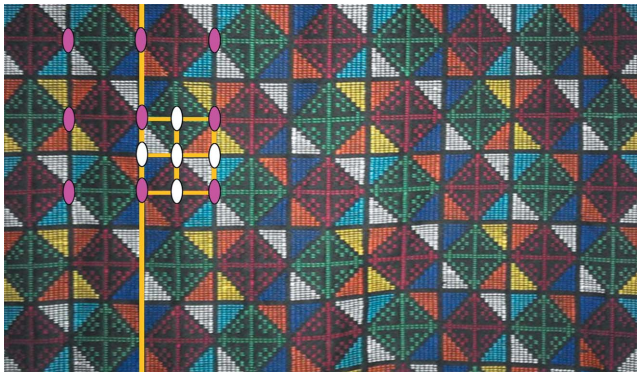


Figure 14
Yakan textile showing pattern of diamonds with symmetry group *p*2*mm* (lattice unit with yellow reflection axes; white/pink centers of rotation). Color group is *p*2 (lattice unit with pink centers of rotation). (*Textile with diamonds*, photographed by Debbie Marie Verzosa, April 2015, Yakan village, Zamboanga City.)

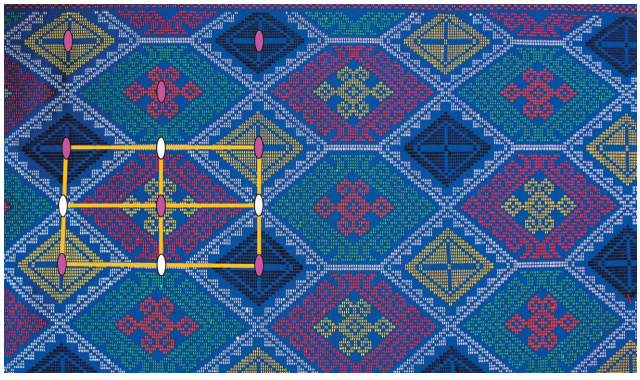


Figure 15
Yakan textile showing *bunga sama* pattern with symmetry group *p*2*mm* (lattice unit with yellow axes of reflection and pink centers of rotation). The color group is *p*2 (lattice unit with pink centers of rotation). (*Bunga sama design from the Phillipine Textile Council collection*, photographed by Agnes Garciano, May 2014, Manila, Metro Manila.)

weft technique. Three black stripes run through the central panel of the blanket, indicating its use for the deceased of high rank in society. The motif in Fig. 11(*c*), common in Bontoc funerary textiles, is referred to as the *matmata*, which means 'eyes of the ancestors', serving as a guide as one travels to the afterlife (De Las Peñas & Salvador-Amores, 2016). Fig. 12(*a*)



Figure 16
Yakan textile showing pattern with symmetry group *p*1*g*1. (*Textile with leaves of a vine*, photographed by Debbie Marie Verzosa, April 2015, Yakan village, Zamboanga City.)



(*a*) (*b*) (*c*)

Figure 17
(*a*) Yakan *saputangan* with finite designs in the center and four corners of symmetry type D_4, and friezes with symmetry group *p*2*mm*; (*b*) and (*c*) show closer views. (*Yakan suputangan from the F. Quintos collection*, photographed by Ma. Louise Antonette De Las Peñas, September 2014, Manila, Metro Manila.)

depicts a Bontoc weaver with her backstrap loom, weaving the central panel of the *pinagpagan*.

4.3.2. Discontinuous supplementary weft. A related technique is the discontinuous supplementary weft, very prominent among the Yakan weavers in southwestern Philippines. In this weave, the additional weft yarns are not woven across the entire length of the warp; they may be cut and be limited to a few warp yarns (Fig. 12*b*). Weft yarns are discontinuous, so it is possible to produce designs or patterns that change colors from left to right across the warp yarns.

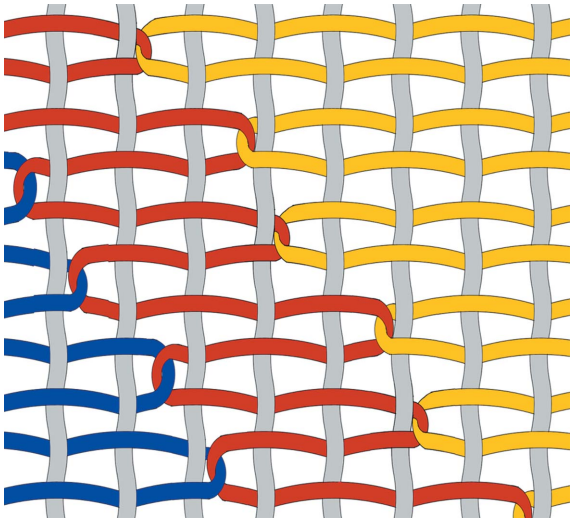
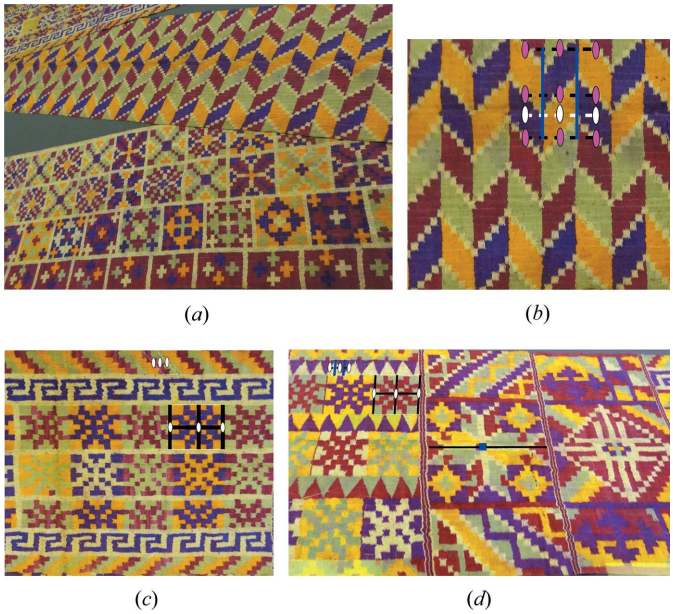


Figure 18
Tapestry weave.

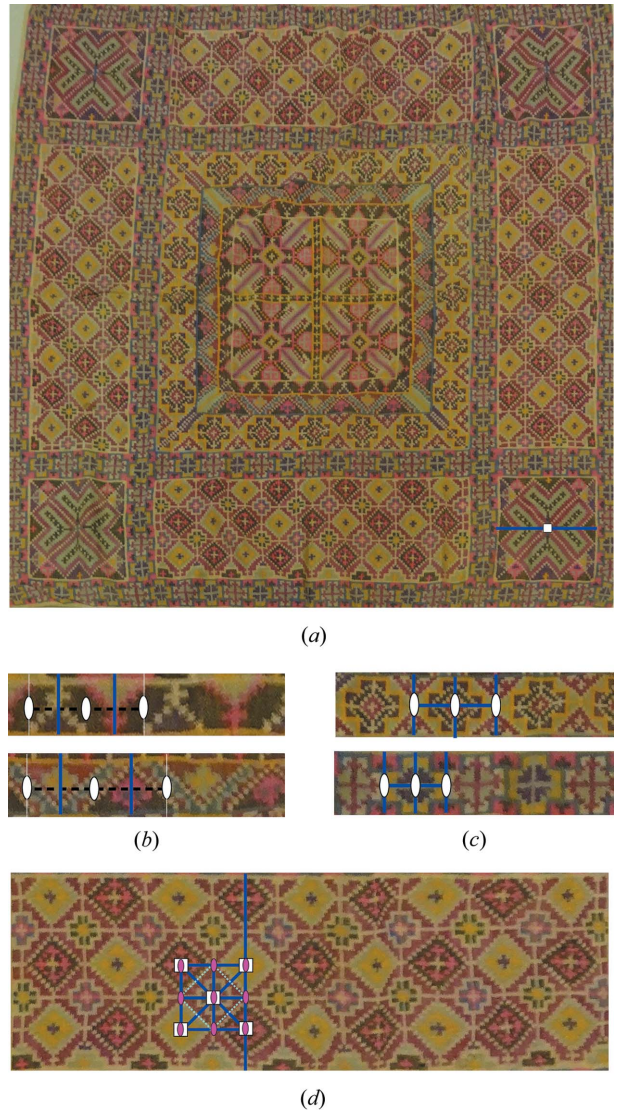


(*a*) (*b*) (*c*) (*d*)

Figure 19
(*a*) Tausug *kandit*; (*b*) plane pattern with symmetry group and color group *p2mg* (lattice unit of symmetry group is half that of color group); (*c*) frieze patterns with symmetry groups *p*211, *p*1 and *p*2*mm*; (*d*) finite design with symmetry group D_4. (*Tausug kandit from the F. Quintos collection*, photographed by Ma. Louise Antonette De Las Peñas, September 2014, Manila, Metro Manila.)

The Yakans employ this weaving technique to create colorful clothing with intricately woven designs. Their traditional clothing includes narrow cut pants resembling breeches, usually with rainbows (*palipattang*) (Fig. 13), and cloth wrapped around the waist by women as a tubular skirt or used by men as a waistcloth where a weapon can be inserted (Figs. 13–16). Traditional motifs include diamond motifs (Figs. 14 and 17), the *bunga sama* inspired by the skin patterns of a python (Fig. 15) and the *dawen-dawen*, meaning leaf of a vine (Fig. 16). The head cloth used by men is called *saputangan* and often shows geometric motifs (Fig. 17).

The Aklanon from Panay, western Visayas, also use the discontinuous supplementary weave in creating patterns in their *piña* textiles, made from the fibers of the leaves of the red Visayan pineapple.



(*a*) (*b*) (*c*) (*d*)

Figure 20
(*a*) Tausug *pisyabit*. Shown in the center and four corners are finite designs with the symmetry group D_4. Frieze patterns with symmetry groups (*b*) *p2mg* and (*c*) *p2mm*. (*d*) Plane pattern with symmetry group *p4mm*; color group *p*2 (lattice unit with pink centers of rotation). (*Tausug pisyabit from the F. Quintos collection*, photographed by Ma. Louise Antonette De Las Peñas, September 2014, Manila, Metro Manila.)

4.3.3. Tapestry. A special kind of woven textile produced using the discontinuous supplementary weft technique is the tapestry weave, where the principal weft itself is discontinuous, as opposed to the supplementary weft. In this weave, the weaver counts warp threads to be lifted so that the principal weft threads can be inserted to create the design or pattern (Gustilo, 2014). To hold the fabric together, weft yarns are dovetailed or locked with those adjacent (Fig. 18) (Rubinstein, 1989). This weaving technique is employed by the Tausugs from southwestern Mindanao; the Tausug men have their own version of the waistcloth called the *kandit* (Fig. 19*a*) and a head cloth called the *pisyabit* (Fig. 20) in tapestry weaves. The *kandit* shown is skillfully woven with a variety of geometric motifs of contrasting colors which combine friezes, plane patterns and finite designs that are characteristic of traditional Tausug textiles. Some weaves of the Bontocs and Yakans also incorporate the tapestry technique.

5. Symmetry structures by weaving techniques

5.1. Decorative dyeing

Vertical reflection symmetry was present in all the *t'nalak* textiles analyzed. The presence of vertical reflection symmetry is a consequence of the way in which the threads are knotted before the dye is applied (Fig. 3*b*). Among the symmetry group types of the plane, the patterns found were *p*1*m*1, *c*2*mm*, *p*2*mm*, *p*2*mg* and *c*1*m*1 (Figs. 2*a*–2*e*), which contain vertical reflections. In some samples, plane patterns are terminated by frieze patterns at the end of the cloth. For instance in Fig. 2(*f*), we see a plane pattern with the symmetry group *p*2*mm* and a frieze pattern with symmetry group *p*1*m*1 at the border of the cloth.

Fig 4 shows an example of a warp *ikat* weave that is one of the traditional textiles in Bagobo society, consisting of several frieze patterns with symmetry groups *p*1*m*1 and *p*2*mm*.

The weft *ikat* tubular garment from the Maranaos that is shown in Fig. 5 consists of several frieze patterns with symmetry groups *p*2*mm* and *p*1*m*1.

5.2. Decorative weaving

The *binakul* is an example of a face-to-face weave, the back portion exhibiting a plane pattern consisting of squares and rectangles. A typical example of the *binakul* with symmetry group *p*2*mm* is shown in Fig. 6(*a*). Fig. 6(*b*) shows another variant of the *binakul* with the quarter circle motif, which has the symmetry group *p*1. Interestingly, this plane pattern has a symmetry group that closely approximates *c*1*m*1, with diagonal reflection axes passing through the corners of the black squares. The repeated patterns in Figs. 8(*a*) and 8(*b*) show the back and front, respectively, of textiles also woven using the face-to-face weave. The plane patterns have symmetry group *p*2*mm*.

In our study, the symmetry analysis was focused on woven textiles showing a finite design or a repeated pattern, arising from a basic unit or motif. For this reason, although plaids and stripes fall under the decorative weaving technique, we do not consider these textiles to exhibit a motif (not a square or a rectangle) and they are not considered in the symmetry analysis.

5.3. Supplementary thread

5.3.1. Continuous supplementary weft. *Pinilian* and *dinapat* styled designs or repeated patterns in Tinguian blankets demonstrate different symmetry group types. For example, Figs. 9(*a*)–9(*c*) show plane patterns with symmetry groups *p*1, *p*1*m*1 and *p*2*mm*, respectively. The patterns in Figs. 10(*a*) and 10(*b*) have the frieze group *p*2*mm* and plane group *p*2*mg*, respectively.

The blanket in Fig. 11 is typical of a Bontoc funerary blanket consisting of three panels, the top and bottom adorned with frieze patterns with the symmetry group *p*2*mm*. Both ends of the central panel are adorned with plane patterns with the symmetry group *p*1*m*1.

5.3.2. Discontinuous supplementary weft. The *palipattang* shown in Fig. 13 consists of layers of frieze with the symmetry group *p*2*mm*. The repeated patterns of the Yakan textiles in Figs. 14 and 15 are planar patterns with the symmetry group *p*2*mm*, while that shown in Fig. 16 has the symmetry group *p*1*g*1. The *saputangan* presented in Fig. 17 highlights a traditional feature in the Yakan head cloth, consisting of diamonds in the center square, adorned with four squares at the corners with identical motifs, and frieze patterns at the sides. The center square and four corner squares adopt the inherent symmetries of the square and have symmetry group D_4. The frieze patterns have the symmetry group *p*2*mm*.

5.3.3. Tapestry. For the *kandit* in Fig. 19(*a*), one side has a plane pattern with the symmetry group *p*2*mg* (Fig. 19*b*). The other side changes pattern from four layers of frieze each with the symmetry group *p*2*mm* [see the lower portion of Fig. 19(*a*)] to seven layers of frieze, with the symmetry groups *p*211, *p*1 and *p*2*mm* (Fig. 19*c*); to eight layers of frieze with the symmetry groups *p*2*mm* and *p*2*mg* [see the left portion of Fig. 19(*d*)]; to finite designs including those with the symmetry group D_4 [see the center of Fig. 19(*d*)].

A typical *pisyabit* is shown in Fig. 20(*a*), consisting of five squares (one center, four corners) and four rectangles with friezes adorning the regions between these quadrilaterals. The squares have symmetry group D_4. The friezes have symmetry group *p*2*mg* (Fig. 20*b*) and *p*2*mm* (Fig. 20*c*). For the rectangular regions, the plane pattern has the symmetry group *p*4*mm* (Fig. 20*d*).

5.4. Discussion

The previous section presented a number of the existing symmetries found in each of the three broad categories of Philippine woven textiles. Also, as indicated in §4, there were 138, 59 and 391 repeated patterns and finite designs analyzed, respectively, for decorative dyeing, decorative weaving and supplementary thread techniques. For each weaving technique, the percentage of patterns and designs that exhibit a given symmetry group type was calculated and will be reported below.

5.4.1. Plane patterns. Given the nature of textiles which are two-dimensional, most patterns include repetition in two dimensions. Out of the 588 patterns and designs studied, 389 were plane patterns and the distribution of these patterns is presented in Fig. 21. Although these percentages are not intended to be estimates for the actual percentages of the entire population of woven textiles, they still show which symmetries are more common in a particular weaving technique.

It is clear from Fig. 21 that, in all of the samples analyzed, threefold and sixfold symmetries are absent, with the non-occurrence of the plane groups *p*3, *p*3*m*1, *p*31*m*, *p*6 and *p*6*mm*. Since the warp and weft threads in the textile are perpendicular, a three- or sixfold rotation does not map a motif onto itself (Grünbaum & Shephard, 1988).

Furthermore, the supplementary thread technique gives the most variety in terms of symmetry. It is possible that the 'freedom' offered by inserting additional threads can broaden the range of designs that may be produced using this technique. For example, the textile from the Yakans showing *dawen-dawen* (Fig. 16) has the symmetry group *p*1*g*1, which is a rare group in our sample. We have not come across a planar pattern with symmetry group *p*2*gg* from the Yakans using the supplementary thread technique, but it is not impossible to create these patterns. Instead, customary preference of patterns such as *bunga sama* or diamond *etc*. gives rise to the occurrence of other planar groups such as *p*2*mm*.

All *ikat* or decorative dye planar patterns in our sample exhibit a vertical reflection which may be a consequence of the culture that produces the patterns, because it is possible to make patterns that have no vertical reflection. However, it is remarkable that the most common symmetry group for the *ikat* is *c*2*mm* (*e.g.* Fig. 2*b*), whereas none of the decorative weave samples and only a few supplementary thread samples produced this symmetry group. Instead, the most common symmetry group for these two weaving techniques is *p*2*mm*. Both *c*2*mm* and *p*2*mm* groups have 180° rotation and reflections. However, while all centers of rotation in a *p*2*mm* pattern are on reflection axes, the same cannot be said of *c*2*mm* patterns (Fig. 22).

It is conceivable that the frequency of a *c*2*mm* symmetry group type in *ikat*, though not in the other types of weaves, is a consequence of the weaving technique. It is customary for the *ikat* patterns in our sample to have motifs that are arranged diagonally following a rhombic lattice. This is possible through the manner the threads are knotted before the dye is applied (Fig. 3*b*). However, this is not easy to achieve in decorative weaves, which demonstrate mostly patterns with rectangular lattices, such as those with *p*2*mm* as the symmetry group. (Figs. 6*a*, 8*a* and 8*b*).

The textiles in our sample that show a 90° rotation, for instance, are the tapestry weaves (see *pisyabit* in Fig. 20). As a matter of fact, it was only among the Tausug fabrics where the plane pattern symmetry group *p*4*mm* was found (Fig. 20*d*). In these examples, both warp and weft threads are of the same width, so that when they are woven perpendicular with respect to one another, a 90° rotation is certainly possible. The relative rarity of 90° rotations and lack of plane groups such as *p*4 and *p*4*gm* may be a result of cultural preference, as weavers may intentionally employ a float weave (Figs. 6 and 8) or use a more prominent thread (Fig. 14), meaning a 90° rotation is unlikely.

5.4.2. Frieze patterns. Out of the 588 patterns and designs studied, 166 were frieze patterns (43 *ikat* and 123 supplementary thread).

Fig. 23 shows that *p*2*mm* is the most common frieze group found among the supplementary thread samples and, to some extent, is quite common in the *ikat* samples. This shows that most frieze patterns have both horizontal and vertical reflection symmetry. The frieze group *p*1*m*1 is more common among the *ikat* samples than the supplementary thread samples. That is, in many *ikat* samples with this frieze group, there is a vertical reflection (which is also found among *ikat* plane

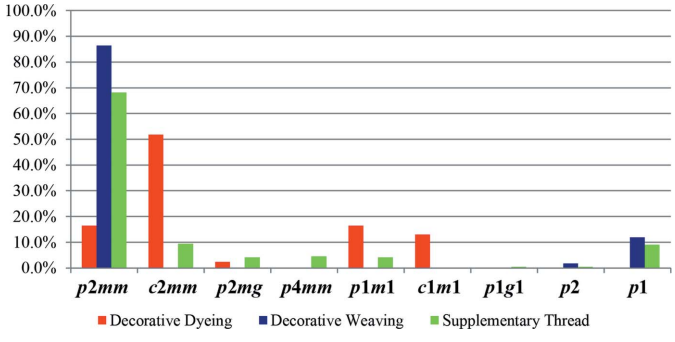


Figure 21
Percentage of each weaving technique exhibiting a particular symmetry group of the repeated pattern.

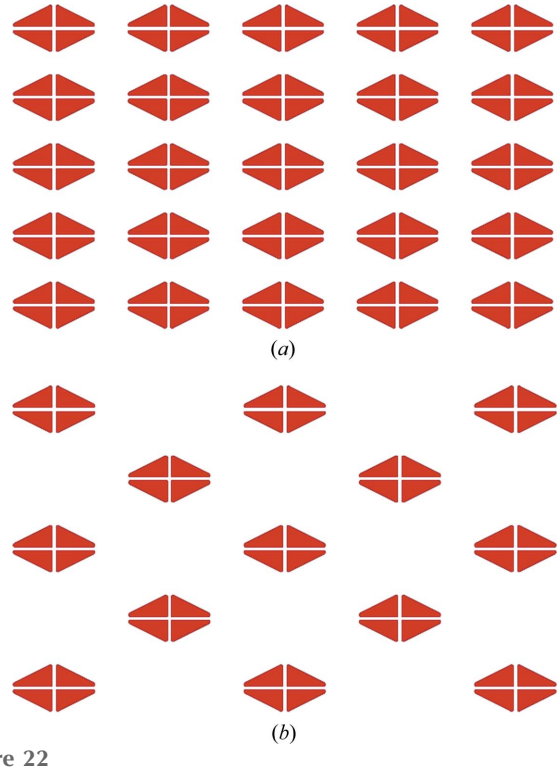


Figure 22
Plane patterns with symmetry group (*a*) *p*2*mm* and (*b*) *c*2*mm*.

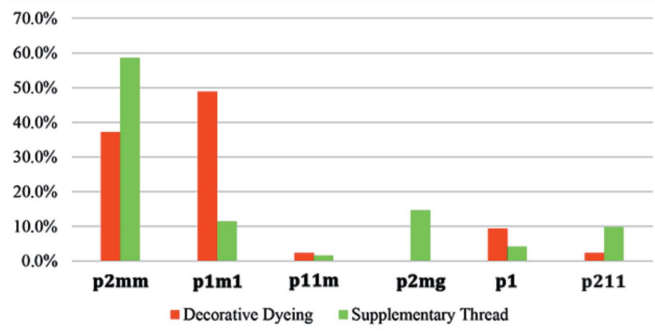


Figure 23
Percentage of each weaving technique exhibiting a frieze pattern.

patterns) but there is no glide reflection or horizontal reflection, or 180° rotational symmetry. The presence of only a vertical reflection is by no means an indication of a limitation in the weaving technique but is possibly a product of the designs employed, which are used to represent the weavers' dreams and connection with *Fu Dalu*, the spirit of the abaca (Paterno *et al.*, 2001).

Frieze patterns with the symmetry group *p2mg* occur in the supplementary thread samples, mostly among the Tausug textile. It is possible for Tausugs (or Yakans) to create friezes with glides using their weaving technique, such as those with the symmetry group *p*11*g*, which is notably not found in any of our samples. Their choice of traditional motifs that are meaningful to their culture leads to friezes with symmetry groups *p2mg* or *p2mm*.

5.4.3. Finite designs. Among the 588 patterns and designs studied, only 33 were finite designs (10 *ikat* and 23 supplementary thread). None of the *ikat* textiles showed a 90° rotation. Weaving a motif with a 90° rotational symmetry will involve a significantly different kind of knotting process than what is done within the weavers' culture. By contrast, 11 out of the 23 (47.8%) supplementary thread fabrics showed a 90° rotational symmetry. These are mostly from the Tausug tapestry weaves such as the *pisyabit* (Fig. 20), whose basic motif elements are squares (De Las Peñas *et al.*, 2014), and this symmetry is also a result of the weaver's choice of threads of the same width.

6. Color symmetry structures

In the symmetry analysis of figures in cultural artifacts, another point of consideration is studying the color symmetries of the designs and patterns. Studies pertaining to color repetition in cultural contexts have been carried out in the literature on San Ildefonso pottery designs (Crowe & Washburn, 1985), sandals of the Basketmaker and Pueblo peoples (Campbell, 1989; Teague & Washburn, 2013), Moorish ornaments (Grünbaum *et al.*, 1986), and Peruvian textiles (Washburn, 1986). Other work on color symmetry of cultural objects has been reported by Makovicky (1986, 2011, 2015, 2016).

6.1. Mathematical considerations

Following Teague & Washburn (2013), we distinguish three levels of color used in woven textiles: one-color designs/patterns, colored designs/patterns and designs/patterns with color symmetry.

'One-color' or 'monochromatic' designs/patterns occur where all motifs are executed in one color against a background of another color (Campbell, 1989), provided that the shape of the background is not the same as the shape of the motif (Teague & Washburn, 2013). In these patterns, the color of the background is not considered. One example is the pattern found in the sides of the central panel shown in the funerary textile of the Bontocs (Fig. 11*c*). The black motifs are set against a white background of different shapes. In this case, the relevant motifs for analysis will consist of the black motifs only.

Both colored designs/patterns and designs/patterns with color symmetry use more than one color. However, in a 'colored' design/pattern, no color change across motifs of the same type occurs. In this case, symmetries of the underlying design/pattern always map a motif of one color to a motif of the same color. In other words, any symmetry of the underlying design/pattern fixes each color. By contrast, designs/patterns with 'color symmetry' occur when a motif of the same shape is assigned more than one color. An example is the pattern of the Yakan fabric in Fig. 14, where triangles are assigned five colors: orange, white, yellow, dark blue and light blue.

A design/pattern with color symmetry can further be identified as a design/pattern with perfect or non-perfect color symmetry. In a design/pattern with 'perfect color symmetry', each symmetry of the uncolored design/pattern permutes the given colors. If a particular symmetry permutes the colors, it is referred to as a 'color symmetry' and it maps all parts of the design/pattern having the same color onto parts of a single color. Colored designs/patterns are generally perfectly colored because a permutation of colors is given by the identity permutation (mapping a color to itself).

Corresponding to every design/pattern with color symmetry is its associated color group. The color group consists of all the symmetries of the uncolored design/pattern that permute the given colors, that is, it is the group containing all the color symmetries. For a design/pattern with perfect color symmetry, its symmetry group is also the color group.

6.2. Color symmetries in Philippine textiles

Colors are perceived and executed in textiles differently, depending on the culture, as we have seen in the colorful designs and patterns of particular Philippine textiles. The T'bolis, for instance, use the colors black, red and natural beige, manifesting their dreams, myths and beliefs in the colored patterns of the *t'nalak*. The Iloko and Tinguians use two shades of a color to create optical repeated patterns meant to ward off bad spirits. The Yakans and Tausugs use colorful motifs to create repeated patterns in celebration of bravery in battle, joy in birth and marriage rituals.

Table 1
Summary of the color groups of textiles with non-perfectly colored designs and patterns.

The figure number of an example included in the count is indicated in parentheses.

Symmetry group	Color group						
Plane group	*c*2*mm*	*p*2	*p*1*m*1	*p*2*mm*	*p*2*mg*	*C*1	Total
*c*2*mm*				1		1	2
*p*2*mm*		2 (Figs. 14 and 15)	2	3 (Fig. 13)		2	9
*p*2*mg*					3 (Fig. 19*b*)		3
*p*4*mm*		2 (Fig. 20*d*)				1	3
							Total: 17

Symmetry group	Color group				
Frieze group	*p*2*mm*	*p*1*m*1	*p*211	C_1	Total
*p*2*mm*	1	4 (Fig. 17*a*, bottom/top frieze pattern)	1	1	7
*p*2*mg*		1			1
*p*1*m*1				1	1
					Total: 9

Symmetry group	Color group					
Finite group	D_2	D_1	C_4	C_2	C_1	Total
D_4	4 (Fig. 19*d*)		1	1	6 (Fig. 17*a*, center square)	12
D_2		1			2	3
						Total: 15

Many of the textiles we have analyzed that employ the decorative dyeing and weaving techniques are colored designs/patterns. A characteristic feature of all *ikat* repeated patterns that we have analyzed is that the motifs of the same kind have the same color, which may be attributed to the dyeing technique. In the *binakul* weave, though the warp and weft each have two hues, the overall effect is a repeated pattern consisting of squares and rectangles, where squares or rectangles of the same size appear to be of the same color. In general, decorative face-to-face repeated patterns such as the one that appears in Fig. 8(*a*) are colored patterns, where a motif is repeated throughout the fabric, without a change in color.

For textiles woven using the supplementary weave technique, there are repeated patterns with color symmetry. In Fig. 9, the patterns have perfect color symmetry since all the symmetries of the underlying symmetry structure effect a permutation of the colors, and are color symmetries. In Fig. 9(*a*)–9(*c*), there are translational symmetries that fix and other translational symmetries that interchange the colors. In Fig. 9(*b*), horizontal reflection symmetries with axes passing through the center of the motifs fix the colors, and those with axes between the motifs interchange the colors. In Fig. 9(*c*), reflection symmetries with vertical and horizontal axes passing through the center of the motifs fix the colors, and the reflection symmetries with horizontal axes passing between motifs interchange yellow and white. The Tinguian blanket in Fig. 10(*b*) displays a repeated pattern with perfect four-color symmetry. The triangle motifs are assigned four colors: red, yellow, white and light green. The reflection symmetries with horizontal axes passing through the vertices of the isosceles triangles whose bases are vertical fix the four colors. On the other hand, the 180° rotational symmetries centered at the points at which pairs of red isosceles triangles intersect fix red and green, and interchange yellow and white. Another example of a pattern with perfect color symmetry is the Yakan textile shown in Fig. 16.

The Yakan and Tausug weavers use colorful repeated patterns in their textiles, with a preference for multi-colored motifs. There are instances where certain colors are used more than others, or used by the weaver in two or more motifs. The use of colored threads in this manner gives rise to patterns with non-perfect color symmetry. For example, consider the planar pattern with seven colors shown in Fig. 14, with the symmetry group *p*2*mm*. The triangle motifs are woven with five colors (dark blue, light blue, yellow, white and orange) and the diamond motifs with two colors (red and green). Note the use of dark blue in every row of triangles. A reflection with the vertical axis passing through the point where red and green diamond motifs meet maps the dark blue to both white and orange, and does not effect a permutation of the colors. The 180° rotations with centers colored pink (shown on the lattice unit of the color group in Fig. 14) effect a permutation of the colors. Here the color group is *p*2. Another example of a planar pattern with non-perfect color symmetry is the *bunga sama* pattern shown in Fig. 15, whose symmetry group is *p*2*mm*, consisting of five colors. The color yellow appears in the centers of the diamond and hexagonal shaped motifs. The reflection with the vertical axis passing through the center of a diamond motif does not effect a permutation of the colors. It fixes the yellow color of the diamond motif, but interchanges the yellow and orange regions of two hexagons. The color group of this colored pattern is *p*2. The 180° rotations with centers colored pink, as shown in Fig. 15, effect a permutation of the colors. The frieze from the Yakan textile in Fig. 13(*b*) also displays non-perfect color symmetry. The reflection with axes marked in white maps green rectangles to both green and orange rectangles. The color group of this frieze is *p*2*mm*, an index seven subgroup in the symmetry group of the frieze; the frieze group is also *p*2*mm*. The centers and axes of symmetry in the lattice unit of the color group are colored pink and blue.

The Yakan *saputangan* in Fig. 17, the *kandit* in Fig. 19 and the Tausug *pisyabit* in Fig. 20 display designs and patterns with non-perfect color symmetry. The finite design (center square) in Fig. 17 has a horizontal reflection symmetry that maps yellow colored motifs to either a yellow or a white colored motif. For this design, the symmetry group is D_4 while the color group is C_1. The planar pattern on one side of the *kandit*

in Fig. 19(*b*) has a 180° rotational symmetry, centered at a yellow parallelogram, that maps yellow parallelograms to either yellow or red parallelograms. The color group for this planar pattern is the plane group *p*2*mg* which is of index two in its symmetry group, also of type *p*2*mg*. The centers of rotation and symmetry axes included in the lattice unit of the color group are colored pink (centers), black (axes of glide reflections) and blue (axes of mirror reflections). In the rectangular region of the *pisyabit* in Fig. 20(*d*), a reflection symmetry with axes passing through the yellow diamond motifs fixes the yellow color of the diamond motifs but maps the yellow inner region of a cross to a pink region. The color group of this colored pattern is *p*2 [centers colored pink are superimposed on the lattice unit of the symmetry group *p*4*mm* in Fig. 20(*d*)].

Table 1 presents the color groups corresponding to the non-perfectly colored designs and patterns arising from the supplementary thread technique that are included within the sample considered in this study. Note that there are patterns and designs in which the color group and the symmetry group of the underlying uncolored pattern are not of the same plane, frieze or finite group type. For example, more than 50% of the friezes with symmetry group *p*2*mm* have the color group *p*1*m*1. As previously noted, the use of the supplementary thread technique allows the weaver to create more varied repeated patterns which are reflected by the range of plane and frieze patterns. The color analysis also suggests a link between this weaving technique and the mixture of patterns with color symmetry. Unlike the decorative dyeing and decorative weaving techniques, the supplementary thread technique allows the weaver to insert (and remove) new colors in the middle of the textile, thereby achieving a wide range of perfectly and non-perfectly colored designs.

7. Conclusion

By no means is this paper exhaustive, showing only a fragment of the designs and repeated patterns found in Philippine culture. Nevertheless, our sample indicates that the types of symmetries and color symmetries produced are related to the weaving techniques and the cultures that produced them.

7.1. Symmetry groups

Our symmetry analysis in this work covers textile patterns and designs that are dictated by the warp and weft strands which are perpendicular, so it is not possible to have threefold or sixfold rotational symmetries. Thus, the groups *p*3, *p*31*m*, *p*3*m*1, *p*6 and *p*6*m* are absent in Fig. 21.

The symmetry groups of the patterns arising from the decorative dyeing technique involve reflection symmetries, namely *c*2*mm*, *c*1*m*1, *p*2*mm*, *p*2*mg* and *p*1*m*1. A large part of the sample have *c*2*mm* and *c*1*m*1 symmetry groups. The *ikat* motifs forming a rhombic lattice may be attributed to the manner in which the knots are tied before dyeing.

On the other hand, the decorative weaving techniques have motifs that form a rectangular lattice; the plane groups mostly present are *p*2*mm*, *p*2 and *p*1. The plane group *p*2*gg* is also possible using this weaving technique, by controlling the manner in which the wefts interlock with the warps at particular intervals through the pedal loom to form the planar pattern. However, these patterns are difficult to achieve in this manner and are uncommon.

The supplementary technique gives rise to a wider range of symmetry groups. These include *p*2*mm*, *c*2*mm*, *p*2*mg*, *p*1*m*1, *c*1*m*1, *p*1*g*1, *p*4*mm* and *p*1. The insertion of weft threads gives more flexibility in terms of the symmetrical patterns that arise. The choice of threads of the same width by Tausug and Yakan weavers gives rise to patterns or designs with 90° rotational symmetry.

For the frieze patterns in the sample, the symmetry groups are *p*2*mm*, *p*1*m*1, *p*11*m*, *p*2*mg*, *p*1 and *p*211. Just like for the plane patterns, most of the friezes arising from the supplementary weaving technique have symmetry group *p*2*mm*. Similar to the case of the plane patterns, the friezes with symmetry group *p*1*m*1 mostly arise from the decorative dyeing technique.

We have not seen plane groups *p*4, *p*4*gm* or *p*2*gg* or the frieze group *p*11*g* among the patterns or designs from the Tausugs or the Yakans, but it is not entirely impossible to create such patterns with these symmetry groups. Cultural preference for traditional patterns or prominent weft threads in the supplementary weave give rise to other plane groups.

7.2. Color groups

While there are colored patterns and designs arising from the decorative dyeing and decorative weaving techniques, patterns and designs with color symmetry are obtained using the supplementary technique, made possible through the insertion of colored threads. The use of multicolored threads (usually five colors and more) by Tausug or Yakan weavers to create colorful motifs as dictated by their culture gives rise to both perfectly and non-perfectly colored patterns and designs. The latter result in varying color group structures, summarized in Table 1.

7.3. Philippine National Commission for Culture and the Arts; mathematics education

The richness and variety of designs and patterns in the textiles of Philippine indigenous communities has been evident throughout this work. It is hoped that this study highlights the technical skill and ingenuity of the weavers and eventually contributes in some way to the preservation of their craft. The Philippine National Commission for Culture and the Arts (NCCA) has been working towards the preservation of weaving as a cultural tradition, and has formed Schools of Living Tradition (SLTs) in weaving communities across the country. In a weaving SLT, a master weaver teaches young weavers the techniques required to weave traditional textile designs.

Finally, our analysis of mathematical structures embedded in Philippine textiles has also formed the basis for the creation of lesson sequences in K-10 mathematics (De Las Peñas *et al.*, 2015) which we have put together with the support of the

NCCA. Specifically, exploratory activities and problem solving strategies rooted in Philippine culture were written for student use, and can be used as examples by teachers in constructing similar activities.

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